EXPLOSION-INDUCED PLANE SHOCK-WAVE ATTENUATION IN A SOLID BODY

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Plane shock-wave attenuation of a solid body induced by the explosion of an explosive layer at the surface is calculated in a quasi-acoustic approximation. The results are compared with experimental data obtained by exploding a 50/50 compound of trotyl and hexogen at the surface of aluminum and brass. Measurements were made by electrocontact and capacitance methods at varying distances from the charge, the maximum distance being ten times the thickness of the charge.

1. The knowledge of shock-wave parameters is required in many engineering applications of explosives and in investigations of materials subjected to considerable pressure. The theory of plasticity shows that the relaxation of an initially compressed material passes consecutively through elastic and plastic stages. Experiment [1] had shown that a similar sequence is observed in processes in which relaxation waves reach the shock front from behind, with the first rarefaction wave moving at the velocity of elastic expansion which corresponds to the pressure behind the front and an amplitude approximately equal to four times the dynamic yield stress.

Previously, the hydrodynamic calculations of explosive-wave attenuation in solid bodies were carried out on the assumption of constant entropy throughout the flow region, including the shock-wave front [2, 3]. In that case, the solution is a simple wave with C_+-characteristics. The Murnaghan equation was used in [3] as the isentropic equation of state, while [2] assumed that the Riemann invariant is of the same form as for a gas and that the relation between the pressure and the speed of sound is defined by the polytropic law. However, equations of state of this kind do not agree with sufficient accuracy with the shock adiabate in the full range of investigated pressures.

Experimental data on shock compressibility are usually represented by the linear dependence between the linear velocity N and the mass velocity u of the shock wave

$$N = \alpha + \beta u. \tag{1.1}$$

Hence

$$p = \rho_0 N u = \rho_0 u (\alpha + \beta u). \tag{1.2}$$

Here p is the pressure and ρ_0 is the initial density of the material.

The problem was solved on the assumption that (1.2) is satisfied not only at the wave front but also behind it. This assumption yields straight-line characteristics in Lagrangian coordinates and is exact for equations of state of a special kind. The comparison of material parameters after its relaxation, as derived from the special equation of state, with those obtained from equations of state defining the behavior of real solid bodies [4, 6] makes it possible to evaluate the upper limit of applicability for the quasi-acoustic method in regions of high pressure. It was shown in [7] that up to pressures of $p \approx 1.5\rho_0\alpha^2$, the error in such calculations is, as in gases, of the order of a few percent.

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2. The problem of plane shock-wave attenuation induced by the explosion of an explosive layer at a solid-body surface was solved in a quasi-acoustic approximation in [7].

The equation of state for the explosion products was chosen in the form

$$p = A\rho^3, \tag{2.1}$$

where the constant A was determined from the Chapman-Jouguet conditions at the detonation front. The entropy increment at the detonation-wave front reflected from the explosives-material interface was disregarded.

The material is assumed to be a perfect fluid for which viscosity, thermal conductivity, and rigidity could be neglected. This assumption is entirely satisfactory in shock loads at pressures considerably exceeding the yield stress of the material. It would be difficult, however, to estimate a priori the error resulting from the application of this assumption in the relaxation region. The assumption of entropy constancy throughout the medium, including the transition through the shock, is tantamount to neglecting the terms $\sim (u/\alpha)^3$ in the solution. The validity of this assumption for low compressions was demonstrated in [7]. With these assumptions the solution is a simple wave with straight C_+-characteristics. In solving this problem, the relationship (1.2) between pressure and velocity of the material was taken to be the same throughout the region of motion, as well as the front. In this case, a solution with straight C_+-characteristics can be obtained in Lagrangian coordinates, without the isentropicity assumption.

Let us write the equation of motion in Lagrangian coordinates

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0. \tag{2.2}$$

Using (1.2), we obtain the equation

$$\frac{\partial u}{\partial t} + f(u)\frac{\partial u}{\partial m} = 0, \tag{2.3}$$

whose solutions are u=const and p=const along straight lines of slope

$$\frac{dm}{dt} = f(u) = \frac{dp}{du} = \rho_0(\alpha + 2\beta u). \tag{2.4}$$

Equation (1.2) coincides with the Riemann invariant to within terms of the order of $(u/\alpha)^3$. A more accurate estimate of the quasiacoustic approximation error cannot be given without comparing the dependence p(u) used here with the Riemann invariant derived from specific equations of state.

There exists an equation of state of a special kind which exactly satisfies (1.2) for the linearity of characteristics in Lagrangian coordinates. Substituting (2.4) into the equations of continuity and energy, we obtain

$$\frac{\partial V}{\partial t} = -\frac{1}{f(u)}\frac{\partial u}{\partial t}, \quad \frac{\partial e}{\partial t} = \frac{p(u)}{f(u)}\frac{\partial u}{\partial t}. \tag{2.5}$$

Equation (2.5) contains only derivatives with respect to t; hence, integrating (2.5) at the fixed point m, we derive the dependence between pressure p, specific volume V, and the specific internal energy e

$$\begin{aligned} p &= \rho_0 u(\alpha + \beta u) \\ V &= V_0\left[\frac{\alpha + (\beta - 1)u_1}{\alpha + \beta u_1} + \frac{1}{2\beta}\ln\frac{\alpha + 2\beta u_1}{\alpha + 2\beta u}\right] \\ e &= \frac{u^2 + u_1^2}{4} + \frac{\alpha}{4\beta}(u - u_1) - \frac{\alpha^2}{8\beta^2}\ln\frac{\alpha + 2\beta u_1}{\alpha + 2\beta u}. \end{aligned} \tag{2.6}$$

The mass velocity u in the rarefaction wave appearing in Eqs. (2.6) is a parameter whose value u_1, at the instant of passing at the fixed point m through the front, represents the entropy function.

Let us compare the behavior of relaxation curves emanating from points along the shock adiabate for the equation (2.6) of state with that derived by other equations of state [4-6]. The dependence of density

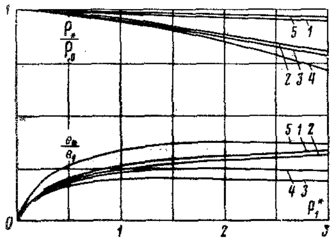

Fig. 1

ρ_* and of specific energy e_* in the final state of relaxation, normalized with respect to density ρ_0 and specific energy e_1 at the front, on the dimensionless pressure $p_1^*=p_1/\rho_0\alpha^2$ at the front is shown in Fig. 1 in the form of curves derived from these five equations of state. Curves denoted by 1, ..., 5 relate, respectively, to the equation (2.6) of state, to those from [4] for $\gamma=3$ and from [5] and [6] for $\gamma=2$.

We note, in the relaxation calculation by these equations of state, stratification of the material into phases was not taken into consideration. This is true for short relaxation times, since the state of the medium in which stratification would have been possible is metastable.

The conformity (within a few percent) of relaxation curves calculated by these equations of state can be observed to extend up to pressures $p_1 \approx 1.5$. Accordingly, we can conclude that condition (1.2) is satisfied with the same accuracy up to such pressures. This defines the upper limit of applicability of the quasi-acoustic calculation method.

3. Let a detonation wave be initiated at instant $t=0$ at point $x=0$ of the charge-free surface. On the above assumptions, the flow throughout the region left of the explosives/medium interface will be defined by a simple centralized wave with straight C_+-characteristics

$$x = (u + c)\, t = wt \tag{3.1}$$

where c is the speed of sound. To find the trajectory of the media interface, at which $dx/dt=u$, we differentiate (3.1) with respect to w along the latter and obtain

$$\frac{dx}{dw} = t + w\frac{dt}{dw} = \frac{x}{w} + \frac{w}{u}\frac{dx}{dw}. \tag{3.2}$$

Hence

$$\frac{dx}{x} = \frac{u\, dw}{w(u-w)}. \tag{3.3}$$

For finding function $u(w)$ we use (2.1)

$$w = u + c = u + (p/A)^{1/3}(3A)^{1/2}. \tag{3.4}$$

From the equality of pressure on both sides of the explosives/medium interface follows

$$p = \rho_0 u(\alpha + \beta u). \tag{3.5}$$

Substituting (3.5) into (3.4), we obtain

$$w = u + \sqrt{3}\, A^{1/6} [\rho_0 u (\alpha + \beta u)]^{1/3}.$$

With A determined from the condition at the detonation front, where $w=D$, we have

$$w = u + b\,[\alpha u(\alpha + \beta u)]^{1/3}, \qquad b = \frac{27}{16}\frac{D}{\alpha}\frac{\rho_0}{\rho_{00}}. \tag{3.6}$$

Here D is the detonation rate and ρ_{00} is the initial density of the explosive. Hence

$$dw = \{1 + {}^1\!/_3\, b\alpha(\alpha + 2\beta u)\,[\alpha u(\alpha + \beta u)]^{-2/3}\}\, du. \tag{3.7}$$

The substitution of (3.6) and (3.7) into (3.3) yields

$$\frac{dx}{x} = -\frac{[\alpha u(\alpha+\beta u)]^{2/3} + {}^1\!/_3 b\alpha(\alpha + 2\beta u)}{b\alpha(\alpha+\beta u)\{u + b[\alpha u(\alpha+\beta u)]^{1/3}\}}\, du = -g(u)\, du, \quad x = \Delta \exp\left(\int_u^{u_0} g(u)\, du\right), \tag{3.8}$$

where Δ is the thickness of the charge. For $x=\Delta$, $u=u_0$. Time t, corresponding to every point x of the interface, is defined by (3.1). Thus (3.8) and (3.1) define the trajectory and the parameters of the solid body at its interface with the explosives, prior to solving the motion problem for the solid body.

4. Let us determine shock-wave attenuation for a solid body with a given dependence u(t) at its boundary. Using (1.2) throughout the flow region, we obtain in mass coordinates a solution with straight C_+-characteristics, along which the mass velocity is "transferred" to the shock front, thus defining its parameters. Let coordinate m=0 correspond to the explosives/medium interface, and t=0 to the beginning of the interface motion. We denote the coordinates of the front by M and T.

The equation of the front and of the C_+-characteristics is then written in the form

$$\frac{dM}{dT}=\rho_0 N, \qquad M=f(t)(T-t)=\frac{dp}{du}(T-t) \tag{4.1}$$

where t is the instant at which the C_+-characteristic leaves the interface. Differentiating (4.1) with respect to t, we consecutively obtain

$$\frac{dM}{dt}=\rho_0 N\frac{dT}{dt}=f'(t)(T-t)+f(t)\left(\frac{dT}{dt}-1\right), \qquad \frac{dT}{dt}=\frac{f'T}{\rho_0 N-f}+\frac{f't+f}{f-\rho_0 N}. \tag{4.2}$$

Using (1.2), we integrate the last equation and obtain the following formulas:

$$T=t+\frac{1}{\rho_0\beta u^2}\int_0^t p\,dt \quad \left(I(u)=\int_0^t p\,dt\right), \tag{4.3}$$

$$M=\rho_0 X=\frac{dp}{du}(T-t)=\frac{\alpha+2\beta u}{\beta u^2}\int_0^t p\,dt=$$
$$=\frac{\alpha+2\beta u}{\beta u^2}\int_0^t \rho_0 u(\alpha+\beta u)\,dt=\frac{\alpha+2\beta u}{\beta u^2}I(u). \tag{4.4}$$

Here X is the Euler coordinate of the front and I(u) is the pressure momentum at the interface at instant of time t at which its velocity was u. Equations (4.3) and (4.4) define the coordinates M and T in terms of the mass velocity u at the shock front which at time t is equal to the velocity of the media interface.

If u is small in comparison with the speed of sound α, from (4.4) we obtain

$$u=\left[\frac{I(u)\alpha}{\beta M}\right]^{1/2}. \tag{4.5}$$

For considerable distances $I(u)\to I_0$, where I_0 is the total momentum imparted by the explosive to the material.

Shock-wave attenuation induced at a metal surface by the explosion of a charge of 50/50 trotyl/hexogen compound was calculated for aluminum and brass by (3.8), (4.3), and (4.4). According to [8, 9] the shock adiabates for these metals are

$N=5.35+1.35u$ (for aluminum), $N=3.76+1.43u$ (for brass).

Instead of the actual shock-wave parameters; $\rho_{00}=1.68$ g/cm^3, $\gamma=2.8$, D=7.65 km/sec, p=266 kbar, and u=2.07 km/sec, the following were used: $\rho_{00}=1.68$ g/cm^3, $\gamma=3$, D=7.95 km/sec, u=1.99 km/sec, and p=266 kbar which, according to [10], define the flow behind the detonation front with adequate accuracy. The calculation results are shown in Fig. 2 in the form of the dependence of the doubled mass velocity W=2u at the front on the normalized distance X/Δ. The curves I and II relate there to aluminum and brass, respectively, and points 1, 2, 3, ..., 6 denote experimental data for Δ=50, 25, 12.5, 5, 3, and 2 mm, respectively.

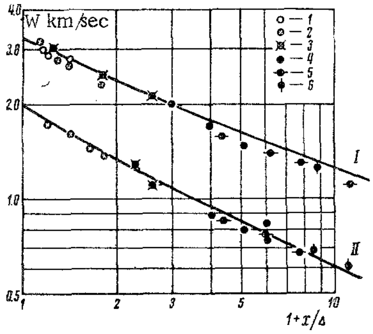

Fig. 2

TABLE 1

x mm	Δ mm	W km/sec
Aluminum AD-1		
3	12.5	3.05
3	25	3.14
7	50	2.98
7	25	2.72
10	50	2.85
10	25	2.63
10	12.5	2.44
20	50	2.74
20	25	2.30
20	12.5	2.11
10.1	5	2.01
10.1	3	1.55
15.1	5	1.70
15.3	3	1.39
15.1	2	1.23
20.5	5	1.42
20.5	3	1.30
20.3	2	1.07
Brass L-59		
5.25	25	1.72
10.5	25	1.63
15.5	25	1.46
20	25	1.38
15.5	12.5	1.30
20	12.6	1.1
10	3	0.856
10	2	0.755
15	5	0.882
15	3	0.82
15	3	0.74
15	2	0.685
20.5	5	0.79
20	3	0.67
20	2	0.604

5. Samples of AD-1 aluminum and L-59 brass of 70 mm diameter and 3-20 mm thick with explosive charges of 60 mm diameter and 2-50 mm thick were used in experiments.

Explosive lenses of 50/50 trotyl/hexogen with a baratol filler were used for generating a plane shock wave by exploding 12.5-50 mm-thick charges. No effect of the exploding lenses on the shock-wave front parameters was observed in samples less than 20 mm thick. The plane detonation wave in 2-5-mm-thick charges was initiated by the impact of 0.08-mm-thick aluminum plates accelerated to a velocity of 5.5 km/sec. In the plate-acceleration device the main explosive mass, consisting of an explosive lens and a 12.5-mm-thick 50/50 trotyl/hexogen charge, was separated from the 3-mm-thick charge of 50/50 trotyl/hexogen by a 3-mm-thick brass screen. The introduction of this screen had cut off the explosion products from the main charge, and made it possible to reduce the explosion-product pressure behind the striker at the instant of its impact on the explosive to 40 kbar. The detonation was generated within approximately 10^{-7} sec from impact. The distortion and slant of the shock front in samples did not exceed 0.5 mm at a diameter of 50 mm.

The speed W of the sample free surface was measured in these experiments directly by electro-contact and capacitance methods. This speed was assumed to be equal to twice the mass velocity of matter at the front. Signals from the capacitance pickup were recorded on an OK-17 oscillograph whose input resistance was equal to the wave resistance of the cable. The speed of the free surface is

$$W = \frac{\varepsilon h}{ERC}\left(1 - \int_0^t \frac{W dt}{h}\right)^2.$$

Here ε is the pickup output voltage; E is the emf of the supply source; h and C are, respectively, the gap and the capacitance of the measuring condenser.

In electro-contact measurements the signals were recorded on two oscillographs with an accuracy of time intervals $\pm 5\times 10^{-9}$ sec. The speed to artificially split 0.1-0.5-mm-thick plates bonded to the samples was measured. The measurement error of these two methods did not exceed ±10% in each experiment. The results of measurements, averaged over two to five tests, are given in the table.

6. A comparison of calculated and experimental results (Fig. 2) shows a satisfactory correlation. The somewhat lower experimental values (~5-10%) for aluminum at considerable distances from the charge may be due possibly to the effect of elastic loading. It is not, however, possible to draw any firm conclusion about this, owing to the simplified form of equations of state for the combustion products. In the investigated pressure range $0.09 \leq p/\rho_0 \alpha \leq 0.45$, considerably exceeding the dynamic yield stress σ of tested materials ($p \gg 10\sigma$), the hydrodynamic approximation is, within the limits of accuracy required in practical applications, suitable not only for defining matter behavior under impact loads but also in the relaxation region.

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